An Open-Source Incompressible Navier-Stokes Solver for Hemodynamic Simulations at High Reynolds Numbers

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INTRODUCTION

Hemodynamic simulations rely on the ability to numerically solve the incompressible Navier-Stokes (INS) equations in realistic geometries. Physiological conditions are often associated to moderate or high Reynolds numbers flow conditions where inertial forces dominate viscous forces. Convection-dominated incompressible flows constitute a challenging class of problems both in terms of numerical stability and computational cost.

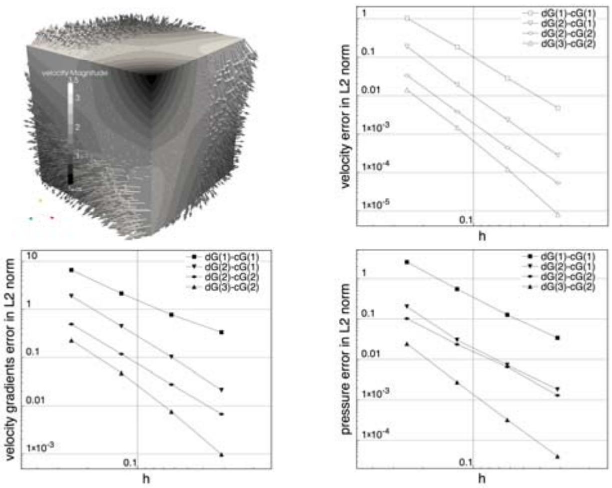
METHODS

An INS solver implementation suitable for large scale hemodynamic simulations in complex three 3D geometries has been recently proposed in [1]. The discrete formulation is based on a pressure-correction scheme combining a discontinuous Galerkin (dG) approximation for the velocity and a standard continuous Galerkin (cG) approximation for the pressure.

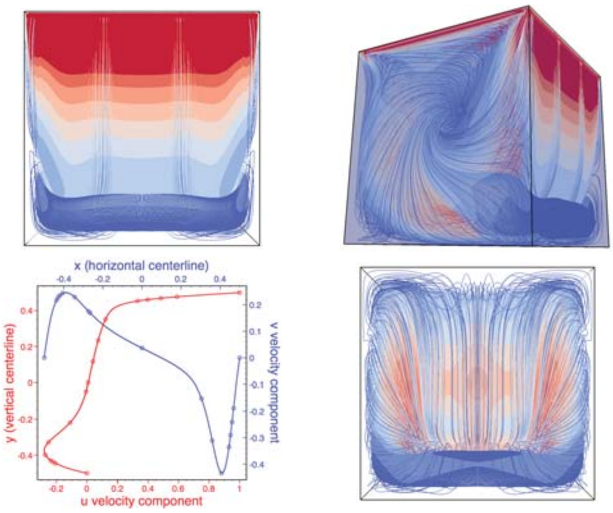
- The main interest of pressure-correction algorithms is the reduced computational cost compared to monolithic strategies.
- The dG discretization of the decoupled momentum equation renders this method suitable for high Reynolds numbers simulations.
- The cG discretization limits the computational cost associated to the Laplacian operator in the projection step.
- The space couple is LBB stable for dG(k)-cG(k-1) velocity-pressure pairs as well as for dG(k)-cG(k) equal order discretizations.

We propose an open-source hemodynamics solver based on this formulation [Gnuid, http://github.com/lorbot/Gnuid].

- The solver relies on libMesh [http://libmesh.sourceforge.net/] for mesh management, finite element function spaces and numerical integration.
- PETSc [http://www.mcs.anl.gov/petsc/] ensures efficiency of the solution process on parallel architectures.

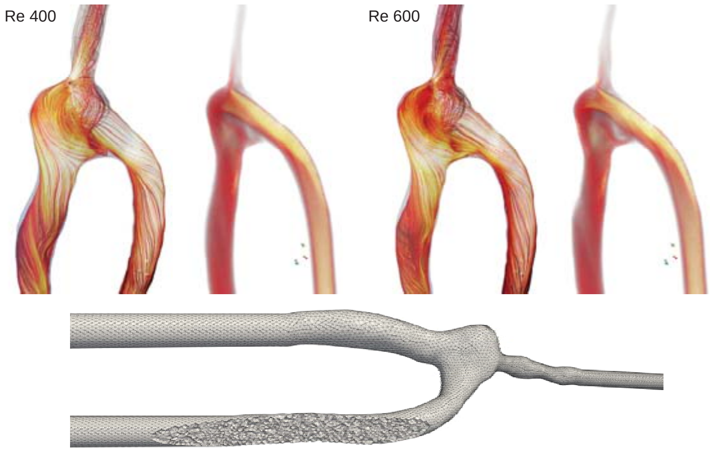


Spatial convergence of the finite element discretization evaluated on the steady counterpart of the Navier-Stokes problem proposed in [2]. We are able to confirm the theoretical convergence rates regarding to the mesh size h.

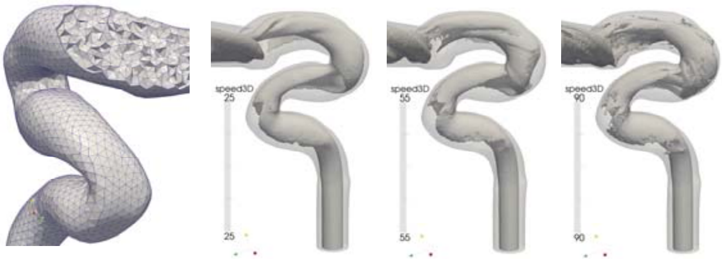


Spatial accuracy evaluated on the 3D lid-driven cavity flow at Re=1000. *Top row and bottom row right*, streamlines. *Bottom row left*, comparison with accurate results presented in [3]. 50x50x50 hexahedral mesh, dG(2)-cG(1) discretization.

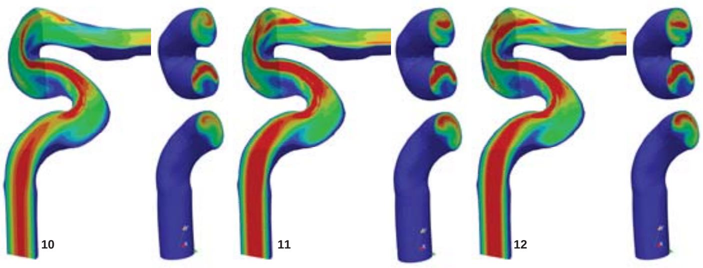
RESULTS

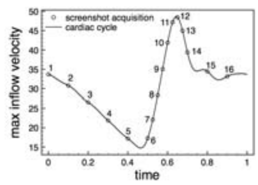


Steady state hemodynamic simulations in an anastomosis surgically created in the arm of an hemodialysis patient. *Bottom,* 253.000 elements tetrahedral mesh obtained with vmtk [www.vmtk.org]. *Top row,* streamlines and velocity volume rendering at Re=400 and 600. dG(2)-cG(1) discretization.



Steady state hemodynamic simulations in a three-bends carotid siphon reconstructed from 3D rotational angiography. *Left,* computational domain and hybrid mesh obtained with vmtk [www.vmtk.org]. *Right,* velocity contours at Re=300, 600 and 1000. dG(3)-cG(2) discretization.





Pulsatile simulation on a three-bends carotid siphon, Re=400, 1000 time steps per cardiac cycle. *Left*, maximum inflow velocity computed by means of the inverse Womersley method [5], and considering a physiological trend over the cardiac cycle. *Top row*, velocity contours over transversal and longitudinal sections of the siphon. 77050 elements hybrid mesh, dg(2)-cG(1) discretization.

CONCLUSIONS

The ability to solve the incompressible Navier-Stokes equations by means of high order accurate finite element methods has been demonstrated. The fully implicit discontinuous Galerkin discretization of the advection-diffusion step extends the applicability of the pressure-correction algorithm to convection-dominated pulsatile flows. The decoupling of the incompressibility constraint from the momentum equation ensures an effective an robust solution process based on standard preconditioned iterative solvers.

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